

Observer Geometry: $R_9 = P_2^2$ and Space Observing Space

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Abstract

The eighth structural level of the spine contributes the rung $R_9 = P_2^2$, the first self-reference on the P_2 -axis. Where $R_4 = P_1^2$ made duality refer to itself, R_9 makes space refer to itself: the spatial axis, held open at level 2, folds back on its own coordinate. The level is centered on Einstein, whose geometric move was exactly the folding of space onto the observer who measures it — space not as a container the observer sits inside but as a structure that depends on how it is read.

1 Standing on Einstein's shoulders

The eighth structural level of the spine is centered on Einstein because Einstein's move was not to add a new dimension and not to discover a new force. It was to fold the spatial axis back on itself and make its local content depend on the observer whose reading was being taken. Before him, space was a container: it held configurations and was itself held by nothing. After him, space was a structure whose local parametrization depended on the frame doing the measuring, and the dependence was not a correction but the substance of the theory. The framework reads this as: space becomes a self-referential coordinate, and the first rung at which the P_2 -axis can refer to itself is $R_9 = P_2^2$.

Einstein was not shunned for his geometry. He was honored for it. What he was shunned for was the later half of his life's work: the refusal to accept that the framework he had opened was the final framework. His insistence that the probabilistic register was incomplete placed him outside the consensus of his own discipline in his later decades, and the consensus treated him as an old man who had not moved with the field. The framework honors him at the rung where his structural move lives, not at the rung where his disagreement with his successors lives.

Remark 1.1 (Geometry as observer-dependent). *The arithmetic content of Einstein's move is the introduction of a coordinate whose value depends on who is reading it. Before this rung, the spatial axis P_2 carries positions that are the same regardless of the reader. At R_9 the axis folds: a position is now a pair, one coordinate for the point and one for the frame of the reader.*

2 What is new at R_9

$R_9 = P_2^2 = 9$ is the second rung whose factorization is a pure square of a prime, the first being $R_4 = P_1^2$. The two rungs are structurally parallel: each introduces self-reference on its own axis. What is new at R_9 is that the self-reference now lives on the *spatial* axis rather than on the duality axis.

Definition 2.1 (Spatial self-reference rung). *The spatial self-reference rung is the smallest positive integer whose factorization is P_2^e with $e \geq 2$. This rung is $R_9 = P_2^2$.*

Proposition 2.2 (R_9 is the first repeated P_2). *R_9 is the smallest positive integer whose factorization contains two copies of P_2 .*

Proof. Integers strictly less than 9 have factorizations in the set $\{1, P_1, P_2, P_1^2, P_3, P_1 \cdot P_2, P_4, P_1^3\}$. None contains two copies of P_2 . The integer $9 = P_2^2$ does, and is therefore the smallest such. $\square$

Key idea

R_9 is to the spatial axis what R_4 is to the duality axis: the first rung on which the axis can read its own coordinate.

3 The folding of the spatial axis

A coordinate that can refer to itself has a second degree of freedom that a coordinate without self-reference does not: the frame in which the coordinate is being read. The spatial axis at level 2 is parametrized by a single number; at R_9 the parametrization acquires a second slot, and the second slot is the observer.

Proposition 3.1 (Two-slot spatial content). *At rung R_9 a spatial configuration is described by a pair (p, f) in which p is the position coordinate on the P_2 -axis and f is the frame coordinate. At rungs below R_9 the frame coordinate is trivial (a constant).*

Proof. At rungs R_n with $n < 9$ the factorization contains at most one copy of P_2 . One copy supports one slot on the P_2 -axis: the position. The second slot, which would carry the frame, has no structural home. At R_9 the second copy of P_2 supplies the second slot, and the frame becomes a coordinate of the configuration rather than a constant property of the reader. $\square$

Remark 3.2 (Frame-dependent content). *The frame slot does not carry an arbitrary piece of information. It carries the answer to the question “relative to what is the position measured.” At R_9 the answer is no longer unique. The first rung at which the frame can be chosen is the first rung at which the choice of frame becomes a structural part of the configuration, and this is the arithmetic content of observer-dependent geometry.*

4 Parallelism with R_4

The rungs R_4 and R_9 are the two smallest nontrivial squares of primes, and they play parallel structural roles on their respective axes.

Proposition 4.1 (Parallel self-reference). *The rungs $R_4 = P_1^2$ and $R_9 = P_2^2$ are the only rungs of the form P^2 for $P \in \{P_1, P_2\}$, and each is the smallest rung on which its respective axis admits a second slot for self-reading.*

Proof. Immediate from Definitions 2.1 and the definition of the first self-reference rung (Article on Level 3, Turing). $\square$

Takeaway

Level 3 (Turing) installed self-reference on the duality axis. Level 8 (Einstein) installs it on the spatial axis. The two installations are parallel and do not interact until the interior point $R_{12} = P_1^2 \cdot P_2$ of the polarity matrix at level 11 (Dedekind) combines them.

5 What is forced at this level

- The spatial axis P_2 admits, for the first time, a self-reading: a configuration at R_9 carries two copies of the spatial coordinate.
- The frame of the observer enters the framework as a genuine coordinate, not as a constant property of the reader.
- The parallel between R_4 and R_9 is established, and the two axes become structurally symmetric with respect to self-reference.
- Every later rung whose content depends on who is doing the reading — which is, eventually, every rung with an interior observer — traces back to R_9 for the first appearance of observer-dependence on the spatial axis.

Key idea

Space becomes observable *from inside itself* at R_9 . Before this rung, space was a container; at this rung, space is a structure that can be read from one of its own positions.

6 Summary

At the eighth structural level the rung $R_9 = P_2^2$ is the first self-reference on the spatial axis. It is the parallel on the P_2 -axis of what $R_4 = P_1^2$ is on the P_1 -axis: the first rung at which the axis can read its own coordinate. The observer enters the framework as a frame coordinate that is no longer trivial, and the spatial content of a configuration is now a pair rather than a single number. The level is centered on Einstein, whose geometric move was exactly the folding of space onto the observer, and whose structural contribution is the first rung at which that fold becomes arithmetically available.