Space:
$P_2 = 3$ and the Opening of the Plane

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Abstract

At the second structural level of the framework, the prime $P_2 = 3$ opens a second dimension and, with it, the possibility of position. The number line alone cannot support here-versus-there; a plane can. This article isolates what is new at the level of P_2: a second axis, a first two-coordinate point, and the arithmetic possibility of a method that examines any claim from more than one position at once. The level is centered on Socrates, who did not answer but opened the space in which answers could be sought, and whose structural contribution is exactly the opening of a plane of inquiry.

1 Standing on Socrates's shoulders

The second structural level of the framework is centered on Socrates because Socrates did not provide answers. He provided the *space* in which answers could be examined. The method of questioning is exactly the opening of a plane: any claim can be interrogated from more than one position, and the positions themselves are the second dimension. At the first level of the framework — the level of duality — a claim is either *yes* or *no*; at the second level, the claim can be examined from here or from there, and the distinction between here and there is the second axis.

Socrates was sentenced to death for opening this plane. The charge was corrupting the youth; the substance was that he made the young examine received claims from positions the received claims had not expected. Opening a plane of inquiry was a structural act that the city did not survive without defending itself against. The framework honors this by placing him at the rung where the plane first opens arithmetically.

Remark 1.1 (Why Socrates and not someone else at P_2)**.** *The level could arguably be held by several figures — any mathematician who first distinguished line from plane, any philosopher who first distinguished thesis from dialogue. The framework chooses Socrates because Socrates is the first person whose* method *is the plane itself, not a result inside the plane. The method is the structural contribution, and the plane is what the method is made of. Other figures stood inside the plane; Socrates opened it.*

2 What is new at P_2

The first level of the framework contributes the prime $P_1 = 2$ and with it the first dimension R^1: a number line, a binary distinction, an on/off, a yes/no. At this level every configuration has exactly one axis, and position on the axis is a single scalar.

The second level contributes the prime $P_2 = 3$ and with it a second dimension R^2: a plane. The arithmetic signature of the opening is the appearance, for the first time, of an integer whose prime factorization requires two distinct primes simultaneously.

Definition 2.1 (First interior point). *The* first interior point *of the framework is the smallest positive integer whose prime factorization contains both $P_1 = 2$ and $P_2 = 3$. This integer is $R_6 = P_1 \cdot P_2 = 6$.*

Proposition 2.2 (6 is the first two-coordinate point). *The integer $R_6 = 6$ is the first positive integer that has a nonzero coordinate on both the P_1-axis and the P_2-axis simultaneously.*

Proof. Every positive integer $n < 6$ has prime factorization either trivial ($n = 1$), a pure power of P_1 ($n \in \{2, 4\}$), a pure power of P_2 ($n \in \{3\}$), or a prime larger than P_2 ($n = 5$). None of these have nonzero coordinates on both the P_1- and P_2-axes. The integer $6 = P_1 \cdot P_2$ has coordinates $(1, 1)$ on the two axes and is therefore the first two-coordinate point. This integer will receive its own treatment at the next composite level of the spine (the level of first synthesis). □

> *Key idea*
>
> At $P_2 = 3$ the number line becomes a plane. Numbers no longer live on a single axis; they have positions, and positions can be compared from more than one direction at once.

3 The method as a structural property of the plane

A plane supports a family of operations that a line does not. Two of them matter structurally for the framework.

Proposition 3.1 (Two operations new at P_2). *A planar configuration admits:*

1. Projection along an axis*: given a point (a, b) on the plane, the projection onto the first axis is a and onto the second axis is b. Both projections exist; neither alone reconstructs the point.*

2. Rotation around the origin*: the plane admits a continuous family of rigid rotations that fix the origin and permute the two axes. The line admits no such family (the only axis-preserving map of a line is the identity and the reflection).*

Proof. Projection along an axis is elementary for a two-coordinate object; for a one-coordinate object the single projection is the identity. Rotation around the origin on R^2 is the one-parameter family $\theta \mapsto R_\theta$, which does not exist on R^1 because a one-dimensional rigid motion is a translation or a reflection, neither of which mixes two axes. □

Remark 3.2 (Socratic method as projection)**.** *The Socratic method is, structurally, projection along an axis. A received claim is examined by projecting it onto a second axis that the claim had not been parametrized by, and the projection reveals whether the claim carries information on that axis or whether its second coordinate is zero. If the second coordinate is zero, the claim was lower-dimensional than it presented itself as, and the examination has located its actual dimensionality. This is the structural content of "Socratic refutation."*

4 What is forced at this level

- A second axis exists. The prime $P_2 = 3$ opens it; nothing in the first level forced its existence.
- A first interior point exists at $R_6 = P_1 \cdot P_2$, and it is the smallest integer with nonzero coordinates on both axes. The level of first synthesis (Aquinas) will treat this point as its own centerpiece.
- Projection and rotation become available as operations. The one-dimensional level admits neither.
- The number line is no longer sufficient. Any framework that stops at P_1 misses the plane; any framework that skips P_2 has nowhere for its first interior point to sit.

Takeaway

$P_2 = 3$ opens the plane. The opening is the prime's entire structural contribution, and the plane is the thing on top of which everything higher will be built. Any later operation that needs two coordinates depends on this level, and any later figure whose contribution uses a plane is standing on the ground that P_2 opened.

5 Summary

At the second structural level the prime $P_2 = 3$ contributes a second dimension, turning the number line into a plane. The first integer with coordinates on both axes is $R_6 = P_1 \cdot P_2$, and it is the smallest two-coordinate point in the framework. The plane supports projection along an axis and rotation around the origin, neither of which was possible at the level of duality. The level is centered on Socrates, whose method is the opening of the plane of inquiry as a structural act. Every subsequent level of the spine depends on this plane existing.