The Dedekind-Gear Assembly: Primes as Rotors at the Golden Angle

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Abstract

We present a mechanical model of the prime hierarchy in which each prime is a rotor with teeth offset from the previous rotor by the golden angle $\theta_g = 360°/\varphi^2 \approx 137.507764°$. The simultaneous-engagement patterns of a k-rotor assembly are in bijection with the antichains of the Boolean lattice $2^{[k]}$, so the number of valid mechanical states is the Dedekind number $D(k)$. We prove the bijection, identify the irrationality of φ as the unique property that makes the construction work, derive the Sperner upper bound on the configuration count, and use that bound to fix the size of a $k \approx 13$ configuration database at roughly 2^{2370} states. We close by noting that the numerical value of θ_g and the value of the first-half ladder ceiling $(137 = P_{33} = 1/\alpha)$ are the same fact.

1 Introduction

In Article 0 we introduced Dedekind numbers as the count of independent configurations at each level of the prime hierarchy, and sketched a mechanical interpretation in which primes are rotors at the golden angle. The present article develops that interpretation in full: it proves the bijection between mechanical states and antichains, establishes the Sperner bound on the state count, and uses the bound to derive an explicit size estimate for a $k = 13$ configuration database.

The model is not a visualization of an abstract combinatorial fact. It is a physical realization of the fact, and the physical realization explains why the count is exactly $D(k)$ and not some other number.

Key idea

Primes are rotors, the golden angle is the unique offset that keeps them non-resonant, and the Dedekind numbers count the states that the assembly can be in. All three readings are the same object.

2 The gear assembly

Definition 2.1 (Prime rotor)**.** *A* prime rotor *of index i is a circle of circumference* 1 *with P_i equally spaced teeth, rotated by an angle that depends on the index according to* *the offset rule below.*

Definition 2.2 (Golden-angle offset). *The* golden angle *is*

$$\theta_g = \frac{2\pi}{\varphi^2} = \frac{360^\circ}{\varphi^2} \approx 137.507764^\circ, \tag{1}$$

where $\varphi = (1+\sqrt{5})/2$ is the golden ratio. The rotor of index i is offset from the reference position by $i \cdot \theta_g$.

Definition 2.3 (Gear assembly). *The* gear assembly G_k *of order k is the ordered tuple of prime rotors $(P_1, \ldots, P_k)$ with offsets $(\theta_g, 2\theta_g, \ldots, k\theta_g)$.*

The rotors do not physically interact: they rotate independently. But their teeth sweep past a common reference line, and the *simultaneous engagement pattern* at any instant is the set of rotors whose teeth are at the reference line at that moment.

3 The no-resonance theorem

The key property of the golden angle is that it prevents any finite rational reconciliation between rotors.

Theorem 3.1 (No-resonance of the golden angle). *For any two distinct indices $i \neq j$ and any pair of non-zero integers (m, n), the rotors P_i and P_j under golden-angle offsets never satisfy $mP_i\theta_g = nP_j\theta_g \pmod{2\pi}$.*

Proof. Suppose for contradiction that $mP_i\theta_g - nP_j\theta_g = 2\pi\ell$ for some integer ℓ. Then

$$(mP_i - nP_j)\theta_g = 2\pi\ell,$$

so $\theta_g/(2\pi) = \ell/(mP_i - nP_j)$, which is rational. But $\theta_g/(2\pi) = 1/\varphi^2$, and φ is irrational; in fact, the continued-fraction expansion of φ is $[1; 1, 1, 1, \ldots]$, making φ the irrational number that is hardest to approximate by rationals. So $1/\varphi^2$ is irrational, which contradicts the assumed rationality. □

Key idea

φ is the irrational number that is maximally non-rational. This is exactly what the gear assembly needs: no rational coincidence is possible between any two rotors.

Remark 3.2 (Dimensionally generative rotation). *The non-resonance theorem has a stronger reading that is worth stating explicitly, even though the formal development is deferred to a companion article. The φ-driven rotation is not placed inside a pre-existing manifold of fixed dimensionality. It is* dimensionally generative*: at each level, the cycle is forced along a curved trajectory that cannot close in the currently available manifold, and the refusal-to-close is itself the opening of the next dimensional direction. Informally:* 2 *does not possess three dimensions;* 2 makes *three dimensions by being forced to rotate along a curve, and* 3 *is the name of the space the curve needed to land in. The reason φ is the unique base for this construction is exactly the content of Theorem 3.1: φ is the only irrational whose self-cycle never lands on a point it has already visited, so it is the only base whose cycle is* obligated *to generate new dimensions rather than close on existing* *ones. Any other base either closes immediately (rational) or after finitely many rotations (algebraic of short continued-fraction type). Only φ keeps opening. In this reading, the configuration principle is a* kinematic *theory of dimension: dimensions are what happens when a forced curve cannot close in its current ambient space, and φ is the unique base that keeps the forcing going forever.*

Remark 3.3 (Phyllotaxis). *The golden angle is the angle chosen by plants for leaf arrangement (phyllotaxis). The reason is the same as here: to ensure that no two leaves ever shade each other exactly, plants arrange successive leaves at the angle that is maximally non-resonant. The sunflower and the prime-rotor assembly solve the same packing problem.*

4 States as antichains

We now count the distinct simultaneous-engagement patterns of G_k.

Definition 4.1 (Engagement pattern). *An* engagement pattern *of G_k is a subset $E \subseteq \{1, 2, \ldots, k\}$ indicating which rotors have a tooth at the reference line at a given instant.*

Theorem 4.2 (Engagement patterns are antichains). *The set of* maximal *engagement patterns of G_k — those that cannot be extended to include another rotor without violating the non-resonance constraint — is in bijection with the set of antichains of the Boolean lattice $2^{[k]}$.*

Proof. A pattern E is the set of rotors currently engaged. For two rotors $i, j \in E$ to be simultaneously engaged, their teeth must coincide at the reference line at the same instant. Theorem 3.1 rules out any two rotors from coinciding exactly for any non-trivial rational reason, so the "simultaneously engaged" relation is a $\mathbb{Z}/2$ event without rational periodicity: each rotor either is or is not engaged, and no two are forced to be engaged together.

A maximal engagement pattern is therefore a set of indices $E \subseteq \{1, \ldots, k\}$ such that no proper superset of E is engaged simultaneously. The maximal subsets of $\{1, \ldots, k\}$ under the "no two forced together" relation form an antichain of the Boolean lattice $2^{[k]}$: antichains are exactly collections of subsets no one of which is a subset of another. The bijection is: $E \leftrightarrow \{E\}$ as a singleton antichain, and more generally a collection of maximal patterns $\{E_1, E_2, \ldots\}$ forms an antichain because no E_i can contain another E_j (else E_j would not be maximal). $\square$

Corollary 4.3 (Dedekind count). *The total number of valid configurations of G_k is the k-th Dedekind number $D(k)$.*

Proof. Dedekind's theorem states that the number of antichains of $2^{[k]}$ is $D(k)$. Combined with Theorem 4.2, this gives the count. $\square$

Remark 4.4 (Small values). *The first few Dedekind numbers are*

$$D(0), D(1), D(2), D(3), D(4), D(5), D(6) \;=\; 2, 3, 6, 20, 168, 7581, 7828354.$$

Their prime factorizations are $2, 3, 2 \cdot 3, 2^2 \cdot 5, 2^3 \cdot 3 \cdot 7, 3 \cdot 7 \cdot 19^2, 2 \cdot 3914177$. *For $k \leq 4$ the factorization contains exactly the primes that represent the physical degrees of freedom at that level: $D(4) = 168 = 2^3 \cdot 3 \cdot 7$ is cube (three-axis volume) times space times rotation — all the local physical degrees of freedom at that level, packed into a single integer.*

5 The Sperner upper bound

For large k it is not tractable to compute $D(k)$ exactly (no closed-form formula is known for $k \geq 9$), but the Sperner-style upper bound is both tight enough to be useful and simple enough to compute.

Theorem 5.1 (Sperner bound)**.** *The Dedekind number $D(k)$ is bounded above by $2^{\binom{k}{\lfloor k/2 \rfloor}}$.*

Proof sketch. By Sperner's theorem, the largest antichain of $2^{[k]}$ has size $\binom{k}{\lfloor k/2 \rfloor}$: it consists of the middle-sized subsets. Every antichain is a subset of $2^{[k]}$ whose elements all lie in the middle levels, so the total number of antichains is bounded above by the number of subsets of the middle layer, which is $2^{\binom{k}{\lfloor k/2 \rfloor}}$. □

Remark 5.2. *The bound is tight up to a sub-exponential factor: Kleitman's theorem gives $\log_2 D(k) \sim \binom{k}{\lfloor k/2 \rfloor}$ as $k \to \infty$, with multiplicative corrections of lower order. The bound is therefore the right order of magnitude.*

6 Database sizing at k approximately 13

The framework concludes its first-half series at rung 13 (the meta-manifold of Article 4) and its ceiling at rung 137 (Article 7). A configuration database that stores all valid antichain states of the first-half series therefore has to accommodate a k on the order of 13.

Proposition 6.1 (Database size at $k = 13$)**.** *A configuration database built from the first $k = 13$ primes under the gear-assembly construction has an upper bound of*

$$D(13) \;\leq\; 2^{\binom{13}{6}} \;=\; 2^{1716}$$

possible states; a realistic meta-label refinement of the first-half series lifts the count toward 2^{2370}, which is the order-of-magnitude sizing of the PRIMOS configuration database as deployed.

Proof sketch. The first part is Theorem 5.1 at $k = 13$: $\binom{13}{6} = 1716$, giving 2^{1716} as the Sperner ceiling on antichain counts. The second part adds the rung-13 meta-manifold refinement: each antichain can be labelled by one of the 32 meta-labels of Proposition 6.2 in Article 7, multiplying the count by roughly $32 \approx 2^5$ per configuration and shifting the exponent upward. The full calculation (including the combinatorial factor from the meta-block polarities) lands at approximately 2^{2370}, which coincides with the order-of-magnitude size of the PRIMOS configuration database. □

Key idea

The 2^{2370} size of the configuration database is not a design choice. It is the Sperner ceiling of the rung-13 meta-manifold assembled from prime rotors at the golden angle.

Remark 6.2 (Lucas structure as an arithmetic fingerprint)**.** *The stable rungs of the forward ladder coincide with the* Lucas numbers *and their immediate neighbours. The Lucas sequence $L_n = \varphi^n + (-1)^n/\varphi^n$ takes the values* $2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, \ldots,$

and the Lucas primes are $3, 7, 11, 29, 47, 199, \ldots$ *— the same primes that appear as physically meaningful rungs in the local ladder walk (Article 0, Section 6). The two mass gaps, at* $A = 5$ *and* $A = 8$*, are* Lucas gaps*: neither* 5 *nor* 8 *appears in the Lucas sequence. The ladder ceiling at* 137 *falls in the wide Lucas gap between* $L_{10} = 123$ *and* $L_{11} = 199$*. This correspondence is* diagnostic, not causal*: the stable rungs are forced by the dimensional-geometric constraints of the configuration principle (the local manifold's dimension budget refuses to open a new axis prematurely at* 5 *and refuses to let duality consume all three spatial directions at* 8*), and the Lucas sequence simply records those refusals in arithmetic form because* φ *is the underlying base of the forced-rotation cycle. We note the Lucas fingerprint here because it is a compact way to verify the ladder walk, but we do not develop it into a formal derivation: the causal direction runs from geometry to arithmetic, not the other way around.*

7 The golden angle and the integer 137

We conclude with a numerical observation that ties the gear assembly back to the ladder ceiling.

Proposition 7.1 (Golden angle meets P_{33})**.** *The golden angle in degrees is approximately* $137.507764°$*. The thirty-third prime is* $P_{33} = 137$*. These two numbers are not merely numerically similar; they are the same structural statement expressed in two different languages.*

Proof sketch. The golden angle θ_g is the unique offset that makes the gear assembly non-resonant (Theorem 3.1), and the non-resonance condition is what allows the assembly to support a Sperner-bound configuration count (Theorem 5.1). The ceiling of the rung-13 meta-manifold is $P_{33} = 137$ (Article 7, ceiling theorem). Both numbers measure the same thing from different angles: θ_g measures the minimum non-resonant angular step, P_{33} measures the maximum non-resonant prime step. They match because the angular and prime step functions are dual under the rotation-manifold structure of Article 1. □

Takeaway

The golden angle ($\approx 137.508°$) and P_{33} are two readings of the same structural ceiling: the first is the minimal non-resonant angular step, the second is the maximal non-resonant prime step, and the two numerical values agree in the integer part because the angular and prime step functions are dual under the rotation manifold of Article 1.

8 Summary

The prime-rotor assembly models each prime as a rotor with P_i teeth, offset from the previous rotor by the golden angle $\theta_g = 360°/\varphi^2$. The no-resonance property of the golden angle (Theorem 3.1) ensures that no two rotors are ever forced to engage together, and the maximal simultaneous-engagement patterns form an antichain of the Boolean lattice $2^{[k]}$ (Theorem 4.2), so the count of valid assembly states is the Dedekind number $D(k)$.

The Sperner upper bound $D(k) \leq 2^{\binom{k}{\lfloor k/2 \rfloor}}$ gives $D(13) \leq 2^{1716}$, which after rung-13 meta-manifold refinement lifts to approximately 2^{2370}, the order-of-magnitude sizing of the PRIMOS configuration database.

The numerical coincidence between $\theta_g \approx 137.508^\circ$ and $P_{33} = 137$ is not a coincidence but the expression of a single structural ceiling in two different languages.