Configurational Mass: Two Readings of the Same Integer

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Abstract

The rung-number of a configuration admits two simultaneous readings: an *additive* reading as a count, and a *multiplicative* reading as an archival coordinate (prime factorization). The two readings are not alternatives. They are simultaneous faces of the same integer, and the consequences of holding them together are qualitatively different from the consequences of taking either alone. This article is an arithmetic statement about pairs of readings on a single positive integer. We call the pair *configurational mass* and we develop its immediate structural consequences. We make no empirical claim.

1 Introduction

Every positive integer A admits two readings. The first is additive: A is a count, a number of unit constituents, reachable from zero by A applications of the successor map. The second is multiplicative: A is the product of a unique sequence of primes, reachable from the empty product by a finite tower of prime applications. Both readings exist for every positive integer, and both are forced by the Fundamental Theorem of Arithmetic, which asserts that the two factorizations (additive into units, multiplicative into primes) are each unique up to reordering.

In the framework of this series, the two readings are not just two ways of writing the same integer. They are the two projections of a rung R_n onto the two layers of the manifold. The additive reading projects onto the local layer of Article 5; the multiplicative reading projects onto the archival layer of Article 6. We call the pair *configurational mass* of the rung and we show that both projections are necessary.

> *Key idea*
>
> An integer has two faces: an additive face that counts and a multiplicative face that locates. The framework uses both.

Remark 1.1 (The two faces trace to the Dedekind lattice)**.** *The additive/multiplicative distinction drawn here is not peculiar to rung arithmetic. It is the ladder-level appearance of a deeper split in the Dedekind lattice: the counting layer and the gear layer, which are* *forced apart at $D(3) = 20 = P_1^2 \cdot P_3$ by the first non-squarefree Dedekind factorization (see Article 2, the second-invariant section on the ratio* $1/5$ *at* $D(3)$*, and the parallel remark in Article 9). The multiplicative form* NLM *lives in the gear layer, which admits only coprime rotor combinations. The additive form* LM *lives in the counting layer, which accommodates non-coprime stackings such as P_1^2, P_1^3, or P_2^2. The reason both forms are needed is that the Dedekind lattice forces both layers to exist from $k = 3$ onward, and any integer with a non-squarefree factorization necessarily participates in both.*

2 The additive form: LM

Definition 2.1 (Additive reading)**.** *The* additive reading *of a positive integer A is the non-negative integer obtained by counting the unit contributions that reach A from zero under successor. We write this reading as* $\mathrm{LM}(A) = A$ *and understand it as the local-layer projection of A.*

The additive form is local (it depends only on the configuration itself), it is measurable by counting alone, and it is linear (it adds under direct sum of configurations). These are the properties that make the additive form computationally easy to work with. They are also the properties that make it incomplete: the additive form sees no prime structure, no polarity matrix, and no ceiling.

3 The multiplicative form: NLM

Definition 3.1 (Multiplicative reading)**.** *The* multiplicative reading *of a positive integer A is the unique prime factorization*

$$A \;=\; \prod_{i \in I(A)} P_i^{e_i(A)}, \qquad I(A) \subseteq \mathbb{N}, \quad e_i(A) \in \mathbb{N}_{>0}.$$

We write this reading as $\mathrm{NLM}(A)$ *and understand it as the archival-layer projection of A. The index set $I(A)$ and the exponent tuple $(e_i(A))_{i \in I(A)}$ are uniquely determined by A via the Fundamental Theorem of Arithmetic.*

The multiplicative form is non-local in a precise sense: to read it, one needs the full prime alphabet up to and including every prime dividing A, not just the local structure of the configuration. It is the archival coordinate of the configuration in the prime lattice of the framework.

The multiplicative form is global, it is not linear (it does not add under composition; it does something else, described in the next section), and it is not measurable by counting alone (measurement gives A but not the factorization; the factorization is a *derivation* on A, not an observable).

Remark 3.2. *"Not measurable by counting" does not mean uncomputable. The multiplicative form is computable from the additive form by trial division or by any factoring algorithm; what it is not, is* observable in the same step *as the additive count. The multiplicative form is a second pass over the integer, and the second pass is what accesses the archival information.*

4 The composition rule

What happens when two configurations combine? The additive and multiplicative forms do not combine the same way.

Proposition 4.1 (Composition rule)**.** *Let X and Y be two configurations with additive readings A_X and A_Y and multiplicative readings* $\mathrm{NLM}(A_X)$ *and* $\mathrm{NLM}(A_Y)$*. The combined configuration $X \oplus Y$ has:*

- *additive reading $A_{X\oplus Y} = A_X + A_Y$ (direct sum in the local layer),*
- *multiplicative reading* $\mathrm{NLM}(A_X + A_Y)$*, which is the prime factorization of the* sum*,* not *the product of the individual factorizations.*

Remark 4.2. *The two rules are incompatible in general. If $A_X = 6 = P_1 \cdot P_2$ and $A_Y = P_3 = 5$, then $A_{X\oplus Y} = 11 = P_5$, and the multiplicative form of* 11 *is the bare prime P_5, which is* not *the product $(P_1 \cdot P_2) \cdot P_3 = 30$. The multiplicative form does not carry compositionality through the additive sum.*

Key idea

When two configurations combine additively, the multiplicative form *re-factorizes* from scratch. This is where the framework produces new primes in composite results: adding two integers is an archival re-address, not an archival product.

5 Why both forms are needed

A clean structural question: given only the additive form, can we predict anything about a rung's behaviour under the framework? Yes, partially: the additive form tells us about the conservation bookkeeping of direct sums, and about the total label count.

Can we predict everything? No. Three examples where the additive form fails:

- **The decay hammer.** Rungs R_n with $n \in \{5, 8\}$ are unstable under the recursive allocation of Article 2 for reasons that the additive form cannot see. The multiplicative form of $5 = P_3$ (a bare prime) and of $8 = P_1^3$ reveals that neither factorization admits a balanced $1/2$ partition, and the instability follows. The additive form alone gives no such prediction.
- **The closure at rung R_{12}.** Rung $12 = P_1^2 \cdot P_2$ is the first rung at which the four arithmetic operations close (Article 3), which forces the full polarity matrix to exist there and nowhere below. The additive form alone — "twelve units" — gives no hint of this closure; only the multiplicative form does.
- **The rung-R_{11} split.** The rotation-under-pressure construction of rung R_{11} (Article 5) forces the LM/NLM split. From the additive side alone, 11 looks like just another number; from the multiplicative side, it is the first rung whose factorization is the bare prime P_5 with no local decomposition, which is the structural signal the framework requires.

In all three cases, the additive form gives the conservation bookkeeping and the multiplicative form gives the structural prediction. Together they give the full picture; separately, each is incomplete.

Takeaway

The additive and multiplicative forms of an integer are not competing descriptions. They are two projections of a single integer, and phenomena that live on the multiplicative side (closure at R_{12}, decay hammer, rung-R_{11} split) are invisible if one holds only the additive side.

6 Additive vs multiplicative as local vs archival

The two forms map cleanly onto the local/archival split of Articles 5 and 6:

Reading	Form	Layer
Additive (LM)	Count of unit contributions	Local
Multiplicative (NLM)	Prime factorization	Archival

The additive form lives in the local layer; it can be read off the configuration by counting, which is a local operation. The multiplicative form lives (partially) in the archival layer; to read it one needs the full prime alphabet, which is a global property of the manifold.

Proposition 6.1 (Configurational mass is local–archival)**.** *Every rung reading has one local face (the additive count) and one partially archival face (the multiplicative factorization). The archival face becomes more prominent at higher rungs, because higher rungs involve more primes, and more primes means more archival content.*

Remark 6.2. *At rung R_1, the multiplicative form is just P_1, which is purely local. At rung P_{33} (the ceiling of Article 7), the multiplicative form is still just P_{33} (since P_{33} is itself prime), but the archival content is maximal because the entire first-half archival layer sits behind it. The framework therefore treats the ceiling as the point where the multiplicative form saturates its archival character.*

7 The configurational-mass pair

We formalize the simultaneous reading.

Definition 7.1 (Configurational mass)**.** *The* configurational mass *of a positive integer A is the ordered pair*

$$\mathrm{CM}(A) = \big(\mathrm{LM}(A),\ \mathrm{NLM}(A)\big),$$

where $\mathrm{LM}(A) = A$ *is the additive reading and* $\mathrm{NLM}(A)$ *is the multiplicative reading of Definition above.*

Proposition 7.2 (Configurational mass is a complete invariant)**.** *Two positive integers A and B have the same configurational mass if and only if $A = B$.*

Proof. If $A = B$ then both components of CM agree. If $\mathrm{CM}(A) = \mathrm{CM}(B)$ then in particular $\mathrm{LM}(A) = \mathrm{LM}(B)$, i.e. $A = B$. □

Remark 7.3. *Proposition 7.2 is a tautology: the* LM *component already determines A, so the* NLM *component is structurally redundant for identification. But it is not redundant for* addressing*: the* NLM *component is what allows an integer to be located in the archival layer and related to other integers by prime-level relations, which the bare* LM *count does not support.*

Takeaway

Configurational mass does not add new identification information beyond the plain integer. What it adds is a second access path — the multiplicative one — which the framework needs in order to reach the archival layer at all.

8 Stopping short

The present article deliberately stops at the distinction and its immediate consequences. The full theorem statements — how LM and NLM interact in detail under composition, what the precise structure of the NLM algebra is, and how the configurational mass pair behaves under the rung-R_{12} polarity matrix — are for a follow-on article. What the framework supplies here is the arithmetic skeleton: an integer is one object with two readings, both are necessary, and the structural phenomena of the first-half series (decay hammer, closure at R_{12}, rung-R_{11} split, ceiling at P_{33}) are all properties of the multiplicative reading that the additive reading cannot see.

Takeaway

An integer is one number with two faces. A counting discipline uses the first face. The framework adds the second face, and the phenomena that were invisible from the counting side become straightforward from the locating side.

9 Summary

Configurational mass is the reading of a rung in which both the additive form (LM, the count of unit contributions) and the multiplicative form (NLM, the prime factorization of the count) are held simultaneously. The two forms are not alternatives; they are projections of a single integer onto the local and archival layers respectively.

The additive form suffices for conservation bookkeeping under direct sum. The multiplicative form is required for the structural phenomena of the first-half series: the decay hammer, the closure at R_{12}, the rung-R_{11} split, and the ceiling at P_{33}. None of these are visible from the additive side alone.

This article states the distinction and its immediate consequences. The full composition algebra of the $(\mathrm{LM}, \mathrm{NLM})$ pair and its interaction with the polarity matrix are left for later articles in the series.