The Local / Non-Local Split: Why Rung 11 Is Forced to Branch

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You can't sum the loops you don't have,
you need to be able to reach them.
— P.H.C.G.

Abstract

Every rung of the prime ladder up to and including rung 10 admits a single arithmetic reading. At rung 11, the ladder is forced to split: the same rung number supports two distinct structural readings, the *local mass* (LM) reading and the *non-local mass* (NLM) reading. The split is not a matter of interpretation; it is a consequence of the fact that 11 is the first rung whose construction requires a path that is not available from "below" in the ladder. We derive the split, identify its two physical faces (the LM face as additive mass number, the NLM face as multiplicative archival coordinate), and show that the split propagates upward: once LM $\neq$ NLM has been forced at rung 11, every subsequent rung inherits both faces.

1 Introduction

In Article 0 we built the prime ladder rung by rung and noted that rung 11 — the crown jewel — is the first rung whose construction cannot proceed by local arithmetic alone. The standard arithmetic path, "combine smaller rungs to reach a larger one," has no realization that stays inside the arithmetic layer for the transition $10 \to 11$. The only available path is the rotation-under-pressure construction ($7 + 4 = 11$, realized physically as ${}^{7}\mathrm{Li}(\alpha, \gamma)^{11}\mathrm{B}$), which produces rung 11 but does so non-locally.

In the present article we take this observation seriously: we show that rung 11 supports two structural readings simultaneously, and that each reading is forced by a different constraint on the ladder.

Key idea

Rung 11 is the first rung that is *both* an additive combination of lower rungs *and* a non-local archival coordinate. Every rung below 11 is only one or the other. Every rung above 11 is forced to be both.

Remark 1.1 (Structural origin: the Dedekind counting layer). *The* LM / NLM *split is visible at rung* 11 *because that is where the nuclear substrate cannot suppress it any longer, but its structural origin is earlier in the Dedekind lattice. The gear-assembly reading of Article 9 holds at most one rotor per prime, so the gear layer can represent only* squarefree *composites. The first Dedekind number whose factorization requires a non-squarefree factor is $D(3) = 20 = 2^2 \cdot 5$; the 2^2 forces the existence of a second layer — the counting layer — in which non-coprime accumulations can live (see Article 2, the second-invariant section on the Pareto ratio at $D(3)$). The* LM / NLM *split of the present article is the nuclear-scale shadow of this older gear/counting split: the additive form* LM *lives in the counting layer, where non-coprime stackings are permitted, and the multiplicative form* NLM *lives in the gear layer, where only coprime rotor combinations exist. Rung* 11 *is simply the first rung at which the shadow is wide enough for the nuclear substrate to be forced to pick a side.*

2 Local and non-local: definitions

Definition 2.1 (Local mass form). *The* local mass form *of a rung n, denoted* LM(n), *is the additive decomposition of n into sums of smaller rungs:*

$$\mathrm{LM}(n) \;=\; \{\, (a_1, \ldots, a_k) \,:\, a_1 + \cdots + a_k = n,\; a_i \in \mathbb{N},\; a_i < n \,\}.$$

The local form is a multiset of additive decompositions.

Definition 2.2 (Non-local mass form). *The* non-local mass form *of a rung n, denoted* NLM(n), *is the multiplicative decomposition of n into products of primes:*

$$\mathrm{NLM}(n) \;=\; \prod_i P_i^{e_i}, \qquad n = \prod_i P_i^{e_i}.$$

The non-local form is the unique prime factorization of n.

For rungs 1 through 10, the two forms are *linked*: any additive decomposition of n can be matched with the multiplicative decomposition via the shared ladder structure. The linking is broken at rung 11.

3 The forced split at rung 11

Theorem 3.1 (LM / NLM split at rung 11). *The smallest rung n at which the local and non-local forms cannot be consistently identified is $n = 11$. For all $n < 11$, at least one local-form decomposition can be matched with the unique non-local-form factorization by a ladder-respecting correspondence. At $n = 11$, no such correspondence exists.*

Proof sketch. Rungs 1 through 10 each admit at least one local-form decomposition whose terms correspond, via the rung-labelling rules of Article 0, to factors of the non-local form. For instance, rung $10 = 2 \cdot 5$ has local-form decomposition $(5, 5)$ or $(2, 3, 5)$ or $(8, 2)$; the decomposition $(2, 2, 2, 2, 2) = 2 \cdot 5$ ties the local and non-local forms by matching the multiplicity of 2 in the additive count with the exponent of 2 in the factorization. At rung 10, the correspondence exists.

At rung 11, the non-local form is the bare prime: 11 is irreducible. The local form must therefore be a decomposition of 11 into terms whose multiplicative recombination equals 11. But 11 has no non-trivial multiplicative decomposition, so no additive decomposition can be matched to it by a ladder-respecting map. The only remaining option is a decomposition via the rotation-under-pressure path $11 = 7 + 4$, which is additive but does not reduce to the multiplicative structure of 11 — because there is no multiplicative structure below the prime itself. The correspondence fails here, and the two forms split. □

Proposition 3.2 (Four sub-labels). *Once the split is forced at rung* 11*, every subsequent rung* $n \geq 11$ *inherits four sub-labels:* $\mathrm{LM}^+(n)$, $\mathrm{LM}^-(n)$, $\mathrm{NLM}^+(n)$, $\mathrm{NLM}^-(n)$*, where the* $\pm$ *refers to the rung-*12 *polarity and LM / NLM refers to the rung-*11 *locality.*

Proof. Inheritance: the split at rung 11 is structural, not contingent, so every rung above 11 carries both the LM and NLM readings. The rung-12 polarity matrix (Article 3) supplies the $\pm$ distinction. Combining the two bits gives four sub-labels per rung. □

4 Two faces of the same rung

The two forms LM and NLM are not two different numbers; they are two different *readings* of the same number.

- The LM reading of n answers: "what sum produces n?" It is additive, it is local (depends only on rungs below n), and it records the unit-count history of how the rung was reached from zero by successor.
- The NLM reading of n answers: "what product produces n?" It is multiplicative, it is non-local (depends on the global prime factorization), and it records the archival coordinate of the rung in the prime lattice of the framework.

For rungs where n is prime, the two readings disagree structurally — the additive form has many decompositions but the multiplicative form has only the trivial one $n = n$. For rungs where n is highly composite, the two readings carry comparable information.

Remark 4.1 (Why 11 and not 2, 3, 5, 7). *Primes smaller than* 11 *are also structurally bare, but the mismatch does not force a split there because the local-form decompositions of those primes are short enough to be exhausted by the single-prime factorization. At rung* 7*, for example, the decomposition* $(4, 3)$ *can be read multiplicatively because* $4 = 2^2$ *and* 3 *are both present in the prime factorization of the ladder through rung* 7*. The* $7 = 4 + 3$ *decomposition is rotation-under-pressure (it produces rung* 7 *via the rotation prime), but the ladder still admits a consistent global reading at and below rung* 10*. At rung* 11*, the rotation path* $11 = 7 + 4$ *is the* only *path, and* 11 *itself carries no sub-structure, so the split becomes unavoidable.*

5 Propagation upward

Once split at rung 11, every subsequent rung carries both readings.

Proposition 5.1 (Upward propagation). *For all* $n \geq 11$*, the ladder supports four simultaneous readings:* $\mathrm{LM}^+(n)$, $\mathrm{LM}^-(n)$, $\mathrm{NLM}^+(n)$, $\mathrm{NLM}^-(n)$*. The polarity signs refer to the rung-*12 *polarity matrix; the LM / NLM distinction refers to the rung-*11 *locality split.*

Proof. The rung-12 polarity matrix of Article 3 is itself built on top of rung 11, so it inherits the split. The rung-13 meta-manifold of Article 4 then packages all four sub-labels into a single 2×2 block and propagates them to all higher rungs by its labelling action (Theorem 3.1). □

Takeaway

The LM / NLM split is a property of the ladder itself, not a choice of interpretation. Once forced at rung 11, it never un-splits.

6 Physical readings

The framework takes no position on physics beyond the arithmetic, but the split admits clean physical shadows:

- LM as **additive coordinate**. The sum of unit contributions that build the rung number from below. Additive, local, directly computable from the ladder history.
- NLM as **archival coordinate**. The prime factorization of the rung number, read as a location in the configuration lattice. Multiplicative, global, accessible only through the full ladder.

The two faces behave differently on prime and on composite rungs. At a composite rung R_n, the LM side carries an additive decomposition with several distinct summands, while the NLM side carries a multiplicative decomposition with several distinct factors. At a prime rung, the NLM side collapses to a single point (the prime itself), while the LM side still admits multiple additive histories. The split exists at every rung R_n with $n \geq 11$, but the two sides carry unequal amounts of information depending on whether the rung is prime or composite.

Remark 6.1. *The split is a purely arithmetic statement about the two readings of an integer — as a sum and as a product. It is forced at rung R_{11} by the rotation-under-pressure argument of the Rung 7 construction in Article 0 (the rotation manifold $7 = 4+3$), and it propagates upward through every subsequent rung.*

7 Summary

Rung 11 is the first rung at which the local (additive) and non-local (multiplicative) forms of the rung number cannot be matched by a ladder-respecting correspondence. The split is forced: any attempt to reach rung 11 by pure local arithmetic fails, and the only successful construction is the rotation-under-pressure path $11 = 7 + 4$, which produces rung 11 but breaks the local / non-local identity.

Once split, the two forms propagate upward: every rung $n \geq 11$ carries both an LM reading and an NLM reading, which the rung-12 polarity matrix refines into four sub-labels $\mathrm{LM}^+, \mathrm{LM}^-, \mathrm{NLM}^+, \mathrm{NLM}^-$, and which the rung-13 meta-manifold (Article 4) packages into a single 2×2 block.