Configuration, Not Derivation

Companion 0: The Ladder, Rungs 6–13 (and Above)

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(notation sanitized in collaboration)

April 2026

Abstract

This companion document formalizes the rung-by-rung construction of the first six primes, from $P_3 = 5$ through $P_6 = 13$, and then extends the ladder past 13 into the archival layer whose physical cells are primordial black holes. It is a sanitization of the author's first-pass handwritten derivation. The load-bearing case is rung 11: the only rung in the local ladder that cannot be reached by forward stacking, and therefore the rung at which the formal split between the local manifold (LM) and the non-local manifold (NLM) is forced.

1 Conventions

We index primes and integer ranks separately:

$$P_1 = 2, \qquad P_2 = 3, \qquad P_3 = 5, \qquad P_4 = 7, \qquad P_5 = 11, \qquad P_6 = 13. \tag{1}$$

Manifolds are written $M_{\omega-k}$ for the k-th inward manifold counted from the outer boundary ω; M_π is the rotation manifold; M_0 is the origin.

Key idea

Addition and subtraction are *intra-manifold* (element-level). The four derived operators—multiplication, division, exponentiation, logarithm—are *inter-manifold* (set-level). They emerge together at rung 12 from a single 2×2 polarity matrix.

2 Rung 5: the first stable atom

Rung 5 is the first rung at which a stable bound configuration of five particles exists: ${}^4\mathrm{He} + e^-$. This fixes time as the fifth degree of freedom, completing the 3+1+1 local count.

The $A = 5$ mass gap (${}^5\mathrm{He}$, ${}^5\mathrm{Li}$ both unstable) is the first structural blockade of the forward ladder.

2.1 Smashing ^{5}He and ^{5}Li: the decay hammer

$A = 5$ is not merely unreached—it is *actively refused*:

$${}^5\mathrm{He} \xrightarrow{\text{decay}} {}^4\mathrm{He} + n, \qquad {}^5\mathrm{Li} \xrightarrow{\text{decay}} {}^4\mathrm{He} + p. \tag{2}$$

Both decays dump you back to rung 4 plus a free nucleon. The ladder is pushed downward every time the attempt is made. The only stable rung-5 configuration is the atomic one, ${}^4\mathrm{He} + e^-$, which lives *outside* the nucleus.

3 Rung 6: the doubling

$$R_6 \;=\; R_3 \cdot R_2 \;=\; 3 \cdot 2. \tag{3}$$

Rung 6 is the first composite rank: space has been doubled by duality. This is the first appearance of multiplicative composition.

4 Rung 7: the rotation manifold

$$P_4 \;=\; 7 \;=\; R_4 + R_3, \tag{4}$$

stacking plus space. At this rung the rotation manifold M_π appears. In sanitized form,

$$M_{\omega-1} \;=\; \pi \;=\; \begin{bmatrix} \varphi & \zeta \\ \zeta & \varphi \end{bmatrix}, \tag{5}$$

where ζ is the off-diagonal rotation parameter and φ is the golden ratio on the diagonal. The manifold M_π mediates the $\pi/2$ rotation between $M_{\omega-1}$ and M_0. The harmonic content is the Riemann-line sum

$$Z \;=\; \sum_{n\geq 1} \frac{1}{P_n^2} \;=\; \tfrac{1}{2}, \tag{6}$$

fixed by the $\pi/2$ symmetry.

Takeaway

Rung 7 is where rotation becomes a first-class degree of freedom. $Z = 1/2$ is not fitted—it is forced by the rotation symmetry at M_π.

5 Rung 8: the second mass gap

Physically $A = 8$ is the second mass gap: ^{8}Be is unstable with a lifetime of roughly 10^{-16} s. Together, the gaps at $A = 5$ and $A = 8$ bracket $A = 11$ on both sides. The two simplest fusion recipes land exactly on the forbidden numbers:

$$^2\mathrm{H} + {}^2\mathrm{H} + e^- \;\mapsto\; A = 5, \tag{7}$$

$$^3\mathrm{H} + {}^3\mathrm{H} + {}^2\mathrm{H} \;\mapsto\; A = 8. \tag{8}$$

The two simplest recipes are exactly the two that nature blocks.

5.1 Smashing ^{8}Be and ^{8}Li: the decay hammer again

At $A = 8$ the hammer falls twice:

$$^8\mathrm{Be} \xrightarrow{\text{decay}} {}^4\mathrm{He} + {}^4\mathrm{He} \qquad (\tau \sim 10^{-16}\,\mathrm{s}), \tag{9}$$

$$^8\mathrm{Li} \xrightarrow{\text{decay}} {}^8\mathrm{Be}^* + e^- + \bar{\nu} \xrightarrow{\text{decay}} 2\,{}^4\mathrm{He} + e^- + \bar{\nu}. \tag{10}$$

Both channels dump all of $A = 8$ back to two copies of rung 4. Any attempt to pass through $A = 8$ en route to $A = 11$ is instantly laundered into a pair of alphas before the next fusion can occur. This is why $11 = 4 + 4 + 3$ is not a path: the $4 + 4$ step refuses to hold.

Key idea

Both mass gaps are *self-erasing*. ^{5}He, ^{5}Li, ^{8}Be, ^{8}Li all decay back to rung 4. The forward ladder isn't merely missing rungs at 5 and 8; it is actively pushed back down every time a builder tries them. This is the decay hammer.

6 Rungs 9, 10: stacking without new primes

Rungs $R_9 = 3^2$ and $R_{10} = 2 \cdot 5$ add no new primes. The triple-alpha bypass

$$3 \cdot {}^{4}\mathrm{He} \longrightarrow {}^{12}\mathrm{C} \tag{11}$$

jumps from rung 4 directly to rung 12, skipping every rank from 5 to 11 in one move. This is the forward ladder's workaround for the two mass gaps—and the reason rung 11 is structurally isolated.

7 Rung 11: the crown jewel

The Configuration Principle

Rung 11 is the first rung in the ladder whose construction requires *non-local mediation*. Every forward addition path is blocked by the mass gaps at 5 and 8 and their decay hammers. The only physical realization of rung 11 is by *subtraction from rung 12* via cosmic-ray spallation. This forces the formal split LM $\neq$ NLM at rung 11.

7.1 Every forward path is blocked

$$11 = 3+8 \quad \text{blocked at } A=8 \text{ (decays to } 2\alpha), \tag{12}$$
$$11 = 4+7 \quad \text{no stable } A=7 \text{ bridge}, \tag{13}$$
$$11 = 5+6 \quad \text{blocked at } A=5 \text{ (decays to } \alpha+N), \tag{14}$$
$$11 = 4+4+3 \quad \text{passes through } A=8, \tag{15}$$
$$11 = 3+3+5 \quad \text{blocked at } A=5. \tag{16}$$

No forward path survives both gaps. Rung 11 is forward-unreachable.

7.2 Backward construction from rung 12

Rung 12 is easy: triple alpha delivers ^{12}C in a single step. Rung 11 is then obtained by subtraction,

$${}^{12}\mathrm{C} \xrightarrow{\text{spall}} {}^{11}\mathrm{B} + p, \tag{17}$$

physically realized as cosmic-ray spallation of carbon-12 producing boron-11. Cosmic-ray spallation is the dominant source of ^{11}B in the universe; the local ladder has no other way to reach 11.

7.3 The LM / NLM split

Because rung 11 has to be reached by running *backward* through the ladder, the formal construction must distinguish the local manifold LM from the non-local manifold NLM:

$$\text{LM}: \quad \Psi^+(x,y,z),\ \Psi^-(x,y,z), \tag{18}$$

$$\text{NLM}: \quad \Psi^+(x_+,y_+,z_+),\ \Psi^-(x_-,y_-,z_-),\ \text{manifold norm preserved.} \tag{19}$$

At every rung $k < 11$ the two sectors coincide. At rung 11 they separate: boron-11 is an NLM path whose image in LM is a prime residue after subtraction.

Takeaway

Rung 11 is the only rung whose existence requires the construction to leave the local manifold and return. The split LM $\neq$ NLM is forced here, not at rung 13. Rung 13 only *closes* what rung 11 opened.

8 Rung 12: the polarity matrix and the four operators

$$M_{\omega-5} = \begin{bmatrix} N^+N^+ & N^+N^- \\ N^-N^+ & N^-N^- \end{bmatrix}. \tag{20}$$

The four sign pairs are the four derived operators:

$$N^+N^+ \leftrightarrow \text{exponentiation}, \qquad N^+N^- \leftrightarrow \text{division}, \tag{21}$$

$$N^-N^+ \leftrightarrow \text{multiplication}, \qquad N^-N^- \leftrightarrow \text{logarithm}. \tag{22}$$

Addition and subtraction remain intra-manifold. The four derived operators live inter-manifold—which is why they could not appear before rung 12: the polarity matrix requires both a local and a non-local sign, and the non-local sign only exists from rung 11 onward.

9 Rung 13: the meta-manifold

Rung 13 is the closure of all local physical degrees of freedom:

$$M_{\omega-6} = \begin{bmatrix} \text{LM}^+ & \text{LM}^- \\ \text{NLM}^+ & \text{NLM}^- \end{bmatrix}. \tag{23}$$

This is the manifold-of-manifolds: each entry is itself a sector, local or non-local, positive or negative.

The Configuration Principle

The local ladder has exactly six primes: $2, 3, 5, 7, 11, 13$. Rungs 2, 3, 5, 7 build the four local degrees of freedom (duality, space, time, rotation). Rung 11 is the first rung that requires non-local mediation. Rung 13 closes the meta-manifold.

10 Rungs ≥ 13: the archival function and primordial black holes

Rung 13 closes the local ladder, but the story does not end there. The meta-manifold and its higher counterparts perform an *archival function*: they are where the universe stores configurations that cannot be held locally.

10.1 The archival boundary

Let the local physical DOF space be LM ∪ NLM, closed at rung 13 by $M_{\omega-6}$. Configurations that exceed the carrying capacity of LM ∪ NLM cannot dissolve—the configuration principle forbids it—but they also cannot be expressed in the local ladder. They have to be *archived*. The archival layer is the first sector strictly outside $M_{\omega-6}$:

$$\mathcal{A} \equiv \{ M_{\omega-k} : k \geq 7 \} \quad \text{(archival manifolds)}. \tag{24}$$

Every prime beyond 13 (17, 19, 23, ...) opens a coordinate inside $\mathcal{A}$.

10.2 Primordial black holes as archival cells

The physical objects that realize $\mathcal{A}$ are *primordial black holes*. Three facts line up:

1. A primordial black hole is configured at a scale where no local ladder step (LM or NLM) can hold its content. Its degrees of freedom live past rung 13.
2. Black-hole interiors are *norm-preserving but coordinate-scrambling*: the manifold norm survives, the spatial coordinates $|x, y, z|$ do not. This is precisely the NLM condition from rung 11, iterated into $\mathcal{A}$.
3. Information is not destroyed (configuration principle); it is archived. The holographic bound states that the archival capacity of a region equals its boundary area in Planck units—the number of digital non-local coordinates the meta-manifold can address.

Under this reading, primordial black holes are not anomalies of the early universe—they are the *stabilizers*. They absorbed the configurations the local ladder could not hold during the first epoch and have been releasing them ever since, at a rate set by the archival coordinate count.

The Configuration Principle

Rung 13 is not a terminus; it is a *membrane*. Past it lies the archival layer $\mathcal{A}$, whose physical cells are primordial black holes. Every prime $p > 13$ opens one archival coordinate. Without this archive the early universe would have had no way to park the configurations it could not immediately ground, and the ladder could not have stabilized.

Takeaway

The six local primes $(2, 3, 5, 7, 11, 13)$ built the house. The primes past 13 built the attic. Primordial black holes are the boxes in the attic. They are not exotic—they are structurally necessary for the ladder to close at all.

Status

This document is a sanitization of a first-pass handwritten derivation. Notation has been unified in `shared/commands/manifold_notation.tex`, rung 11 sits where the physics places it, and the ladder has been extended past 13 into the archival layer.